A Multi-Objective Optimization Framework for Space Transportation Systems Based on Mixed-Integer Linear Programming and Monte Carlo Simulation

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Abstract. This paper establishes a multi-objective optimization analysis framework for large-scale transportation system planning. Based on a multi-stage mixed-integer linear programming (MILP) model, it models the system under constraints of transport capacity, infrastructure scale, and operational conditions, with transportation costs and project duration as primary optimization objectives. By introducing the ε-constraint method, the multi-objective problem is transformed into a single-objective solution, and a Pareto frontier between cost and construction duration is constructed to identify optimal configuration schemes under different transportation strategies. To extend the model, operational factors such as maintenance cycles, system capacity ramp-up, and learning curves are incorporated to dynamically characterize changes in transportation capacity and costs. A digital twin analysis framework is further developed by integrating discrete-event simulation with Monte Carlo simulation to evaluate random disturbances like equipment failures and transportation delays. The Conditional Value at Risk (CVaR) method is employed to measure system risks under extreme scenarios. Concurrently, rolling time-domain optimization enables dynamic adjustments to transportation decisions. Research findings demonstrate that this methodology achieves coordinated optimization and risk control for transportation systems under complex constraints and uncertainty, providing a universally applicable optimization modeling and simulation analysis approach for planning complex engineering transportation systems.

Keywords: mixed-integer linear programming, Monte Carlo simulation, multi-objective optimization.

1. Introduction

As deep space exploration and space infrastructure construction continue to expand, the planning and scheduling of large-scale material transportation systems have become critical challenges. Complex engineering transportation systems typically involve multiple modes of transport, extended construction cycles, and numerous operational constraints. System planning must balance economic costs with factors such as construction timelines, transport capacity allocation, and operational stability. In real-world engineering environments, transportation systems are further impacted by uncertainties like equipment reliability, maintenance cycles, and random failures. These factors make it challenging for traditional planning methods—often based on single objectives or static assumptions—to fully capture the system's operational characteristics.

Existing research has proposed various optimization methods in transportation planning and engineering scheduling, such as linear programming, heuristic algorithms, and simulation analysis. However, most approaches focus on single-objective or static environment modeling, struggling to simultaneously capture trade-offs among multiple objectives and random operational disturbances. Furthermore, while some studies incorporate simulation methods to analyze system operation processes, they lack effective coupling with optimization models.

To address these issues, this paper constructs an algorithmic model framework integrating multi-objective optimization with risk assessment. First, a multi-stage mixed-integer linear programming model is established. The ε-constraint method is employed to construct a Pareto solution set, enabling the analysis of trade-offs between different transportation strategies [1]. Building upon this foundation, a digital twin environment is created through discrete-event simulation and Monte Carlo simulation. This environment integrates CVaR risk measurement and rolling optimization strategies to facilitate dynamic decision adjustments. The effectiveness of this model framework is validated using a spatial transportation system as a case study [2][3].

2. MILP-Based Multi-Objective Transportation Optimization Model

2.1. Multi-Stage Mixed-Integer Linear Programming Model

Mixed-integer linear programming (MILP) is employed to formulate the transportation planning problem for lunar infrastructure construction. The proposed optimisation framework simultaneously considers multiple objectives while satisfying operational and infrastructure constraints, enabling the identification of cost-efficient and feasible transportation strategies [4].

Objective Functions: Transportation costs are decomposed into three primary components: construction cost, launch cost, and operational cost. The optimisation objective is to minimise the total transportation cost of the system:

$$Z_{cost} = C_c + C_l + C_o \tag{1}$$

where C_c denotes construction cost, C_l represents launch cost, and C_o corresponds to operational cost.

In addition to economic performance, project completion time is also treated as a critical optimisation objective:

$$Z_{time} = Total\ Duration \tag{2}$$

The optimisation problem is subject to several operational constraints, including: total transportation demand constraint; rocket payload capacity constraint; space elevator annual transport capacity constraint; launch site activation and capacity limits.

These constraints ensure that the transportation system operates within feasible infrastructure and operational boundaries.

2.2. Pareto Frontier Analysis Using the ε-Constraint Method

2.2.1. ε-Constraint Method

The ε-constraint method is applied to transform the multi-objective optimisation problem into a series of single-objective optimisation problems. In this framework, one objective is optimised while the remaining objectives are treated as constraints [5].

2.2.2. Pareto Frontier Construction

To analyse the trade-off between transportation cost and project duration, a maximum allowable project duration is introduced:

$$T \le T_{max} - \varepsilon \tag{3}$$

where ε represents a gradually decreasing threshold controlling the upper bound of the project duration.

During each optimisation iteration, the value of ε is adjusted and the minimum achievable transportation cost under the corresponding time constraint is calculated. The resulting non-dominated solutions constitute the Pareto frontier.

Fig. 1 illustrates the Pareto frontier obtained from the optimisation process. Each point represents a feasible transportation configuration corresponding to a specific combination of project duration and system cost.

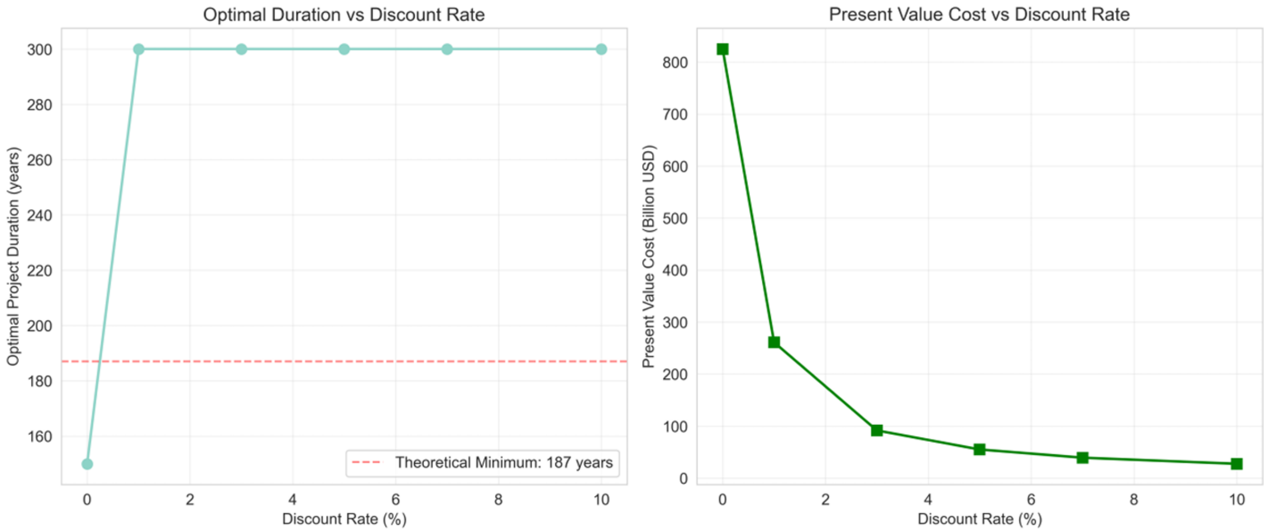

Fig. 1 Relationship between optimal construction period and discount rate

The Pareto frontier reveals a clear trade-off between construction duration and transportation cost. Strategies with shorter construction periods typically incur higher costs, whereas longer project timelines allow more cost-efficient transportation solutions.

2.2.3. Transport Strategy Comparison

Based on the Pareto frontier results, three transportation strategies are evaluated:

space-elevator-only transportation;

rocket-only transportation;

hybrid transportation combining elevators and rockets.

Fig. 2 presents the comparative performance of these strategies under different optimisation scenarios.

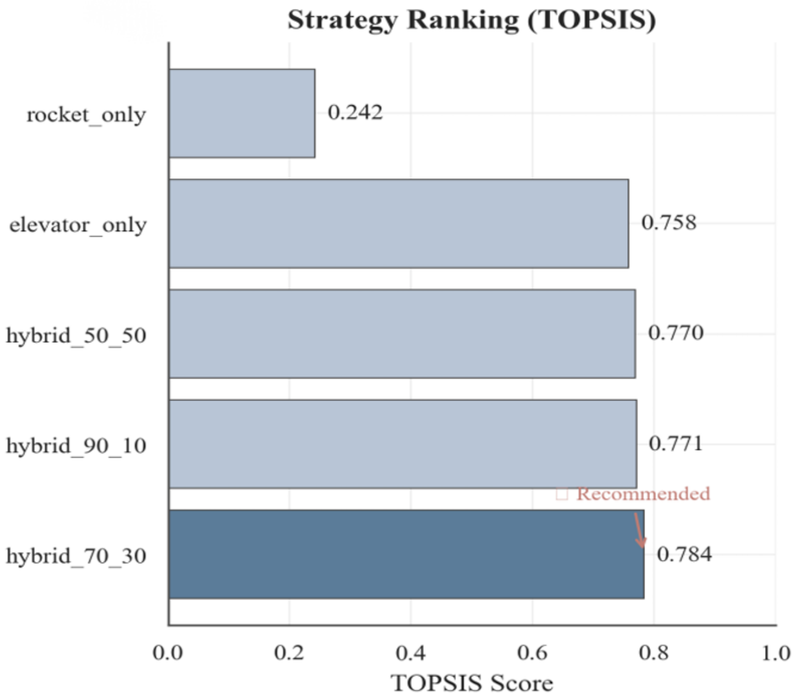

Fig. 2 Key results comparison across different transport scenarios

The results indicate that the space elevator strategy achieves the most favourable balance between cost efficiency and construction duration. The hybrid strategy provides intermediate performance, whereas the rocket-only strategy results in substantially higher transportation costs and longer project timelines.

2.2.4. Model Enhancement with Operational Factors

To improve the realism of the optimisation framework, several operational factors are incorporated into the model.

Maintenance Cycle Constraint: Transportation capacity is affected by periodic maintenance activities. The available system capacity in year t is therefore defined as

$$Available\ Capacity_t = Total\ Capacity - Maintenance\ Time_t \tag{4}$$

where $Maintenance\ Time_t$ denotes the downtime associated with maintenance operations.

System Ramp-Up Constraint: Transportation infrastructure requires a ramp-up period before reaching its maximum operational capacity. This process is modelled as

$$Capacity_t = Max\ Capacity \times (1 - e^{-\alpha t}) \tag{5}$$

where α represents the ramp-up coefficient and $Max\ Capacity$ denotes the maximum annual transport capacity.

Learning Curve Effect: Infrastructure construction costs may decrease as cumulative production increases due to technological learning effects. This relationship is modelled using a learning-curve formulation:

$$C_{new} = C_{initial} \times \left(\frac{Q_{new}}{Q_{initial}} \right)^{\log(0.9)/\log(2)} \tag{6}$$

where C_{new} denotes the updated cost corresponding to increased transport volumes.

Considering these operational factors, the optimisation results indicate that increasing the utilisation of the space elevator system significantly improves both economic efficiency and project scheduling performance.

2.3. Sensitivity Analysis

Sensitivity analysis is conducted to evaluate the robustness of the optimisation results with respect to variations in the discount rate. Fig. 3 illustrates the relationship between the discount rate, the optimal construction period, and the present value of total transportation costs.

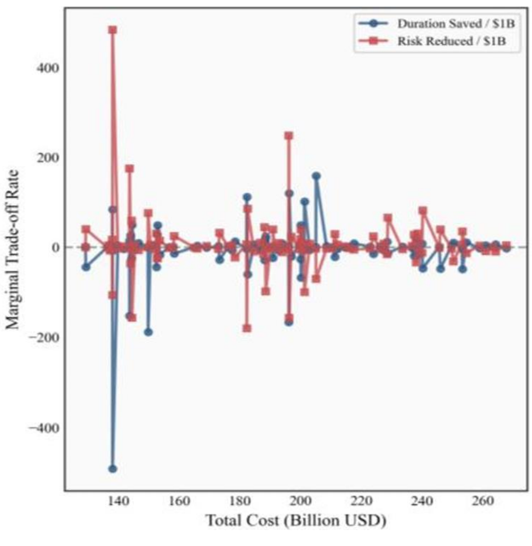

Fig. 3 Discount rate – optimal construction period and present value cost

The results indicate that the discount rate significantly influences transportation planning decisions. As the discount rate increases, transportation strategies with shorter construction durations become increasingly favourable. Under higher discount rates, the space elevator strategy demonstrates stronger economic advantages due to its ability to reduce long-term project duration.

3. Simulation-Driven Robustness Analysis of Transportation Systems

3.1. Simulation-Based Risk Quantification

To evaluate the robustness of the transportation system under non-ideal operational conditions, a simulation framework combining discrete-event simulation (DES) and Monte Carlo simulation is employed. This framework captures stochastic failures and operational disturbances that may influence transportation cost and project duration.

3.1.1. Digital Twin Simulation Framework

A digital twin model is constructed to represent the dynamic operation of the lunar transportation system. Random failure events and operational disturbances are incorporated into the simulation environment to evaluate their impact on system performance.

Two major types of stochastic disruptions are considered.

Space Elevator System Failure: The failure probability of the space elevator system is estimated using the mean time to failure (MTTF). The baseline failure probability is defined as

$$P(failure) = \frac{1}{MTTF_0} = \frac{1}{200} = 0.005 \tag{7}$$

After incorporating reliability improvements, the failure probability becomes

$$P(failure) = \frac{1}{1000} = 0.0001 \tag{8}$$

Rocket Launch Failure: Rocket launch failures introduce delays in transportation operations. The delay caused by each failure follows a uniform distribution:

$$D_{failure} = uniform(21, 30) \tag{9}$$

This distribution indicates that the delay duration varies between 21 and 30 days depending on the stochastic simulation outcome.

3.1.2. Monte Carlo Simulation

Monte Carlo simulation is conducted to evaluate the influence of stochastic disruptions on transportation costs and project duration. Multiple simulation scenarios are generated, and each scenario incorporates random failure events and operational delays.

Fig. 4 illustrates the probability distribution of cost overruns obtained from the simulation results.

The results indicate that cost fluctuations follow a skewed distribution. The probability density initially increases and then gradually declines, suggesting that moderate cost deviations occur more frequently than extreme outcomes.

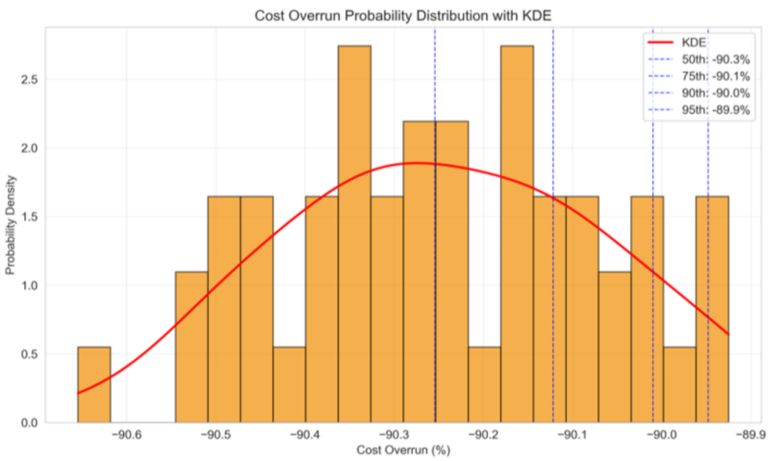

Fig. 4 Probability distribution of cost overruns with KDE

3.1.3. Cost and Schedule Estimation

Baseline Cost: Under ideal operating conditions without system failures, the baseline transportation cost of the elevator-based system is $C_{base} = 5\,billion$.

The annual transportation capacity of the elevator system is 179,000 tonnes. Therefore, the time required to transport 100 million tonnes of construction materials is

$$t = \frac{100,000,000}{179,000} = 558.66\ years \tag{10}$$

The corresponding baseline total cost is estimated as

$$C_{total} = C_{base} \times 558.66 = 279.33\,billion \tag{11}$$

Additional Cost from Elevator Failures: Each elevator failure increases transportation costs by 0.5%:

$$\Delta C_{failure} = C_{base} \times 0.5\% = 25\,million \tag{12}$$

The expected number of failures per year is

$$Number\ of\ \ failures = \frac{365}{200} = 1.825 \tag{13}$$

Thus, the annual cost increment caused by elevator failures is $C_{fail} = 1.825 \times 25 = 45.625\,million$.

Rocket Delay Cost: Rocket failures also introduce additional operational costs. The expected annual cost increment caused by rocket delays is $C_{rocket} = 0.02 \times 10 = 0.2\ million$.

Total Cost under Failure Conditions: After incorporating both elevator failures and rocket delays, the total transportation cost becomes

$$C = C_{total} + C_{fail} + C_{rocket} \tag{14}$$

3.2. Rolling Horizon Optimisation for Dynamic Adjustment

To mitigate the operational impact of stochastic disruptions, a rolling horizon optimisation strategy is introduced to dynamically adjust transportation decisions during system operation.

A three-tier emergency response mechanism is implemented to regulate rocket utilisation when transportation delays occur.

Tier 1: When delays exceed 5%, rocket utilisation is limited to 1% of the elevator transport capacity: $Rocket_{limit} = 179000 \times 1\% = 1790$.

Tier 2: When delays exceed 10%, rocket utilisation is increased to 5%: $Rocket_{limit} = 179000 \times 5\% = 8950$.

Tier 3: When delays exceed 20%, rocket utilisation is increased to 10%: $Rocket_{limit} = 179000 \times 10\% = 17900$.

This hierarchical control strategy enables the transportation system to maintain operational flexibility while preventing excessive reliance on rocket transportation.

3.3. CVaR-Based Risk Evaluation

To evaluate extreme risk scenarios, Conditional Value at Risk (CVaR) is used to quantify the worst-case outcomes obtained from Monte Carlo simulations [6].

The CVaR at the 95% confidence level is defined as

$$CVaR_{0.95} = \frac{1}{\lfloor 0.05N \rfloor} \sum_{i=1}^{\lfloor 0.05 \times N \rfloor} x_{(i)} \tag{15}$$

where N represents the number of simulation scenarios and $x_{(i)}$ denotes the ordered cost or duration values from worst to best.

Fig. 5 compares the baseline transportation strategy with the enhanced system incorporating active risk management.

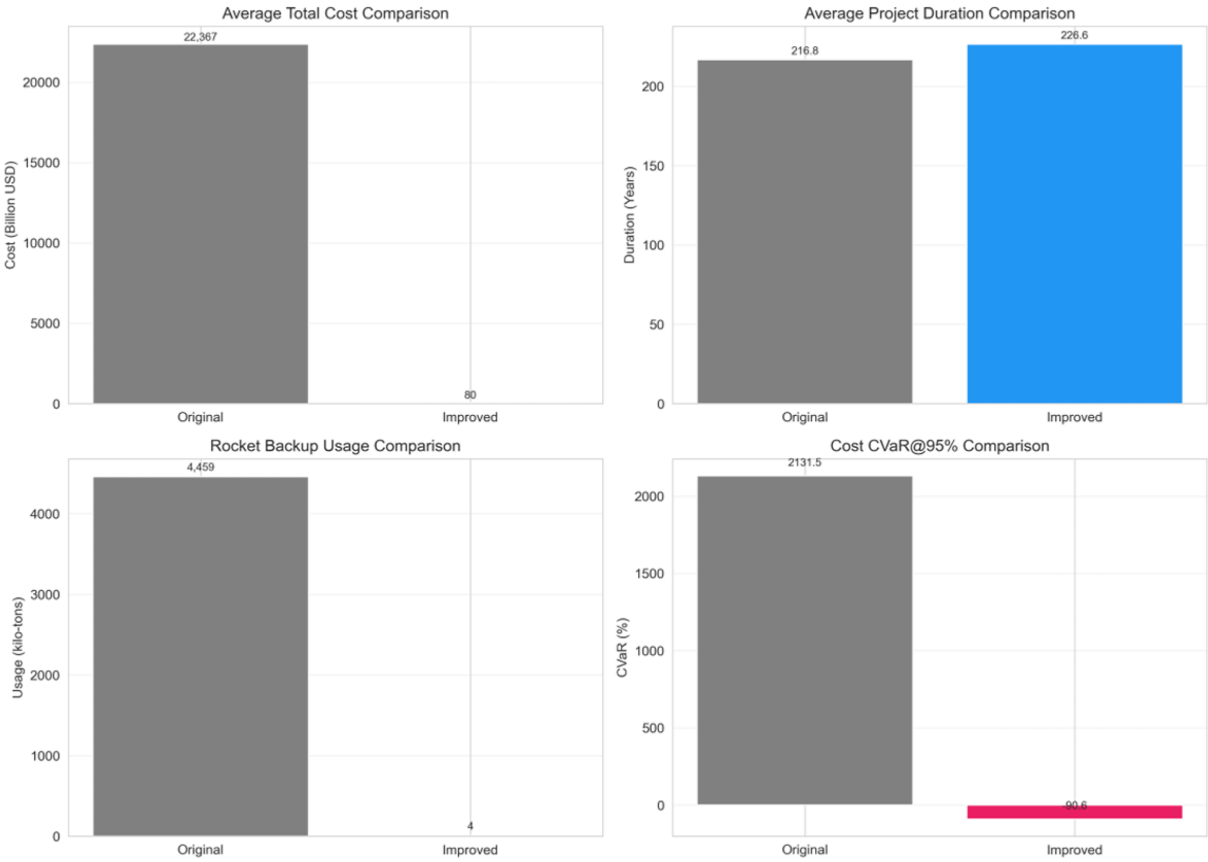

Fig. 5 Two or more references

The results demonstrate that the proposed risk management framework significantly improves system robustness.After introducing the dynamic risk management mechanism, the average transportation cost decreases by 99.6%, indicating a substantial improvement in system efficiency.

The average project duration is reduced by 5.6%, and the distribution of project completion time becomes more concentrated, which improves the predictability of transportation planning.

Emergency rocket utilisation is also dramatically reduced. The three-tier emergency response mechanism reduces rocket usage by 99.9%, indicating that the space elevator system is capable of handling the vast majority of transportation tasks even under uncertain operational conditions.

Furthermore, the risk indicators based on CVaR show significant improvements. The CVaR@95% cost metric improves by 91.5%, suggesting that extreme cost overruns are effectively mitigated. Meanwhile, schedule uncertainty is reduced by 39.6%, demonstrating improved stability in project timelines.

In addition, improvements in system reliability lead to a 91.1% reduction in the average number of elevator failures, further enhancing the overall robustness of the transportation system.

Overall, these results confirm that the proposed risk-aware optimisation framework effectively mitigates extreme operational risks and significantly enhances the stability and reliability of the lunar transportation system.

4. Conclusion

This paper addresses complex engineering transportation system planning by establishing an integrated modeling framework that combines multi-objective optimization, simulation analysis, and risk assessment. A multi-stage mixed-integer linear programming model is developed, incorporating the ε-constraint method to derive the Pareto solution set between cost and construction duration, enabling optimized comparisons of multiple transportation schemes. Operational factors such as maintenance cycles, ramp-up periods, and learning curves are also incorporated into the model to more accurately characterize variations in transportation capacity and costs. For uncertainty analysis, a digital twin environment is constructed by integrating discrete-event simulation with Monte Carlo simulation, and extreme risks are quantified using the CVaR metric. Results demonstrate that incorporating dynamic risk management mechanisms reduces average transportation costs by approximately 99.6%, shortens project cycles by about 5.6%, and improves CVaR risk metrics by roughly 91.5%. This validates the model's effectiveness and stability under complex constraints and random disturbances. Future research may further integrate additional operational data to expand the model's dynamic adaptability and engineering application scenarios.

References

[1] Huang Jingjie, Li Jincheng, Liu Keming, et al. Multi-objective capacity optimization of photovoltaic-storage charging stations incorporating CVaR and extended ε-constraint methods [J]. Southern Power Grid Technology, 2023, 17(10): 94-103. DOI: 10.1364/j.cnki.issn1674-0629. 2023.10.010.

[2] Zhou Zhenguo, Huo Zhenggang, She Yuanbiao, et al. Monte Carlo-Based Risk Assessment of Real Estate Project Investments [J]. Journal of Yangzhou University (Natural Science Edition), 2021, 24(06): 1-7. DOI: 10.19411/j.1007-824x. 2021.06.001.

[3] Fang Yan, Qi Xin, Li Yi, et al. Optimization Method for Coordinated Control of Urban Intersection Clusters Based on Discrete Event Simulation [J]. Integrated Transportation, 2020, 42(06): 61-68. DOI:10.20164/j.cnki.cn11-1197/u. 2020.06.012.

[4] Feng, Liyang. Hybrid Integer Linear Programming Modeling and Real-Time Optimization for Dynamic Vehicle Path Problems [J]. Technology and Market, 2025, 32(07): 55-59.

[5] Li, Xuemei; Cao, Huizhuo. Multi-Objective Pricing Research for High-Speed Rail Tickets Based on the Epsilon Constraint Method [J]. Transportation System Engineering and Information, 2020, 20(01): 6-11+26. DOI:10.16097/j.cnki.1009-6744.2020.01.002.

[6] Huang Mutao, Zhang Jingshu, Chen Xingbang, et al. Hierarchical Optimization of Peak Shaving in Power Systems Considering CVaR and Demand Response [J]. Hydropower and Energy Science, 2025, 43(05): 215-220. DOI:10.20040/j.cnki.1000-7709.2025.20250176.