# The establishment of ski jumping model based on particle swarm optimization algorithm and ODE solver

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**Abstract.** In ski jumping, studying to determine a reasonable run-up posture can effectively reduce air resistance during the run-up phase, thereby increasing sliding speed. However, most current research focuses on kinematic analysis from video perspectives, with fewer studies examining dynamics. Therefore, this paper proposes a maximum speed take-off model based on particle swarm optimization and ODE solver from a dynamic perspective. First, a dynamic model is established using Newtonian mechanics principles, incorporating dynamic factors for attitude control and determining relevant initial parameters. Then, the sliding process is simulated using an ODE solver, and the optimal attitude control parameters are determined using particle swarm optimization. The final model concludes that squatting during the run-up significantly reduces resistance and increases take-off speed. By further considering aerodynamic effects and refining the model by introducing real-time dynamic data, this model can become a core tool for training in winter sports research.

**Keywords:** Particle Swarm Optimization Algorithm, ODE Solver, Ski Jumping.

## 1. Introduction

Ski jumping is one of the most attractive events in winter sports. In the process of completing the event, the take-off speed in the run-up stage is crucial to the final flight distance and score. Through reasonable posture adjustment, athletes can effectively reduce air resistance and increase acceleration in the run-up stage, thus improving the sliding speed. In the existing research, Cao Fengrui et al. (2022) also pointed out that flight distance is an important factor to determine the performance of ski jumping by using the test analysis of athletes' biomechanics, and the take-off stage creates the initial conditions for early flight stage [1]. However, whether it is Tang Weidi et al. (2023) collecting physical morphology data of male athletes in China's ski jumping training team and establishing an aerodynamic digital twin model [2] based on this, or Li Jianyu et al. (2025) filming the take-off phase movements of 16 athletes within a fixed range during competitions and using an artificial intelligence 3D motion analysis system to automatically analyze the captured videos [3], both are merely as pointed out by Zou Jinglun et al. (2024). In existing ski jumping research, there has been relatively detailed exploration of the relationship between take-off and mid-air flight key technologies and athletic performance, as well as their impact on performance or flight distance. However, most analyses focus on the kinematic aspects of video analysis, with fewer studies from a dynamic perspective, especially in aerodynamics [4]. Although three-dimensional analysis of athletes' external movements can also provide relatively effective suggestions for posture control, the relevant conclusions do not reveal deeper dynamic reasons. This paper, however, introduces posture control factors more simply, using particle swarm optimization algorithms and ODE solvers to clearly explain from a dynamic perspective why athletes adopt certain postures to increase speed.

The main contributions of this paper include: 1) the simple introduction of a posture control factor, avoiding complex computational fluid dynamics (CFD) simulations; 2) adopting a dynamic perspective for analysis, revealing the deeper reasons for acceleration during the run-up phase; 3) using particle swarm optimization and ODE solvers, this is the first time such an approach has been applied to analyze the ski jumping process.

The structure of this paper is as follows: Chapter One is the Introduction, which introduces the research background, contributions, and the overall structure of the paper; Chapter Two covers relevant theories, explaining particle swarm optimization algorithms and common methods for solving ordinary differential equations (ODEs). The third section is the experimental design, describing the experimental ideas and procedures. The fourth section presents the experimental conclusions and analyzes them accordingly; the fifth section is the summary, elucidating the significance of the study and potential future directions for expansion.

## 2. Related Theories

Particle Swarm Optimization (PSO) is a global optimization algorithm that mimics the foraging behavior of natural bird flocks. It is a type of swarm intelligence algorithm proposed by James Kennedy and Russell Eberhart in 1995. In PSO, each "particle" represents a potential solution. Particles continuously update their position and velocity to find the optimal solution. The goal of the algorithm is to gradually find the optimal solution through the collaborative efforts of individuals within the swarm. PSO is based on two core principles: 1) Local Learning: Each particle remembers the best solution it has found historically (i.e., the individual optimum); 2) Global Learning: Particles also adjust their search direction based on the best solutions found by all particles in the swarm. Through these two mechanisms, particles continuously adjust their positions, gradually approaching the global optimum [5].

Ordinary differential equations (ODEs) are defined as: any equation containing parameters, unknown functions, and the derivatives (or differentials) of these unknown functions is called a differential equation. Sometimes it is simply referred to as an equation. When the unknown function is a function of one variable, the differential equation is called an ordinary differential equation. The highest order of the derivative of the unknown function in a differential equation is referred to as the degree of the differential equation. The definition of a constant differential equation is as follows:

$$F(x, y, y', y'', \cdots, y^{(n)}) = 0 \tag{1}$$

Ordinary differential equations (ODEs) are typically solved using analytical solutions and numerical solutions (analytical and numerical solutions), with the following distinctions: Analytical or exact solutions (exact solution) fully satisfy the original ODE, but in practice, many mathematical models cannot be solved precisely. In such cases, the only option is to use a numerical solution (numerical solution) to approximate the exact solution. Using an ODE solver is one method for numerically solving ODEs, known as [6]. Figure 1 shows two solutions.

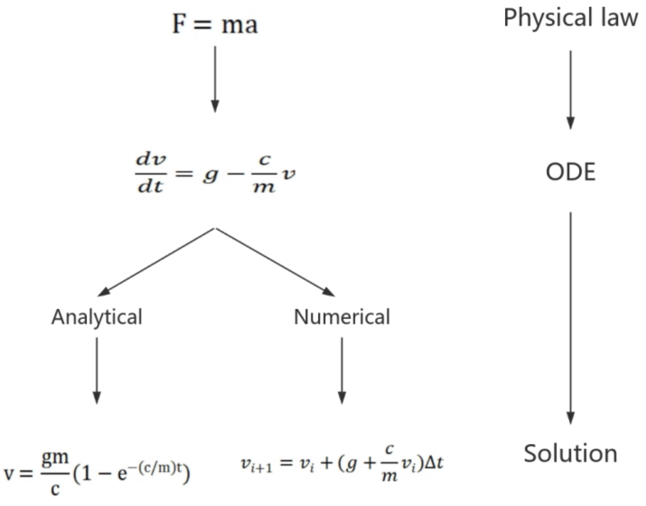

**Figure 1.** Two solutions to a first-order differential equation

## 3. Experiments

In this study, the paper will establish a mathematical model that includes various factors affecting an athlete's sliding speed and convert them into mathematical expressions. These factors include, but are not limited to, slope, athlete mass, friction between the ski board contact surface and the snow, air resistance coefficient, and the athlete's posture. The ODE solver will be used for solving, and then the particle swarm optimization algorithm will be employed to obtain the optimal posture. Figure 2 is a simple experimental flow chart.

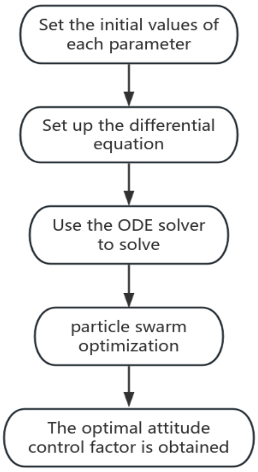

**Figure 2.** Experimental design flow chart

In order to analyze the acceleration process of athletes on a landslide in depth, this paper establishes a mathematical model from the perspective of classical mechanics and fluid mechanics. The sliding speed of athletes will be affected by gravity, friction and air resistance [7]. This paper takes the angle of the ramp as 12.

The component of gravity along the slope is one of the main reasons for the acceleration of athletes. The calculation formula is:

$$F_g = mg\sin\theta \tag{2}$$

$F_g$ is the component along the slope and $\theta$ is the angle of the ski run.

Friction will have a certain hindering effect on the movement of athletes, its expression is:

$$F_f = \mu(\alpha)mg\cos\theta \tag{3}$$

Among them, $F_f$ is the friction force along the slope, and $\mu(\alpha) = \mu_{max}(1 - k_\mu\alpha)$. $\alpha$ varies with different postures of athletes. When standing, $\alpha = 0$; when squatting completely, $\alpha = 1$. $k_\mu$ is the coefficient of influence of posture on friction force, and $0 \le k_\mu \le 1$.

Air resistance is also one of the key factors affecting the athlete's glide, and its formula is:

$$F_d = \frac{1}{2}C_d(\alpha)\rho A_p(\alpha)v^2 \tag{4}$$

Among them $C_d(\alpha) = 1.2 - 0.5\alpha, A_p(\alpha) = A_{max}(1 - k_A\alpha), 0 \le k_A \le 1$. Considering the principle of fluid mechanics, the air resistance coefficient when squatting is obviously smaller than that when standing, so the dynamic representation method [8] is adopted, and $C_d(\alpha) = 1.2 - 0.5\alpha$ is taken. At the same time, the windward area when squatting should be smaller than the windward area when standing, so that $A_p(\alpha) = A_{max}(1 - k_A\alpha)$.

According to Newton's second law, the acceleration of the athlete is given by the following formula

$$a = \frac{F_g - F_f - F_d}{m} \tag{5}$$

Substituting the above expression into (4), we get:

$$a = g\sin\theta - \mu(\alpha)g\cos\theta - \frac{1}{2}\frac{\rho A(\alpha)C_d(\alpha)v^2}{m} \tag{6}$$

This is the formula for the acceleration of an athlete on a slope [9].
A suitable rewrite yields:

$$\frac{dv}{dt} = g\sin\theta - \mu(\alpha)g\cos\theta - \frac{1}{2}\frac{\rho A(\alpha)C_d(\alpha)v^2}{m} \tag{7}$$

also because

$$\frac{dv}{dt} = \frac{dx}{dt}\frac{dv}{dx} = v\frac{dv}{dx} = \frac{1}{2}\frac{dv^2}{dx} \tag{8}$$

Therefore, we can get:

$$\frac{1}{2}\frac{dv^2}{dx} = g\sin\theta - \mu(\alpha)g\cos\theta - \frac{1}{2}\frac{\rho A(\alpha)C_d(\alpha)v^2}{m} \tag{9}$$

This is a first order differential equation.

In this paper, the posture of athletes is determined by $\alpha$, and the particle swarm optimization algorithm is used to optimize the posture parameter $\alpha$. Each $\alpha$ represents a different degree of body bending. In this paper, it is hoped that the optimal take-off speed [10] can be obtained by adjusting the posture configuration.

The fitness function of particle swarm optimization is:

$$f(\alpha) = -v(\alpha) \tag{10}$$

Among them, $v(\alpha)$ is the final velocity under the given posture configuration. The goal is to maximize $v(\alpha)$, so the optimization goal is to minimize $-v(\alpha)$.

At the same time, since various parameters change with $\alpha$, this paper cannot use static formulas but instead employs an ODE solver to more accurately simulate the acceleration process of athletes on the ramp. That is, in each iteration, $A, \mu$, and $C_d$ are calculated based on $\alpha$, followed by ODE solving, and then the particle swarm optimization algorithm is used to obtain the final optimal speed corresponding to $\alpha$.

## 4. Results

After 50 iterations, the final results showed that when $\alpha=1$, that is, when the athlete fully squats, the jump speed of the athlete reaches the maximum value.

Figure 3 shows the relationship between velocity and $\alpha$ of different postures:

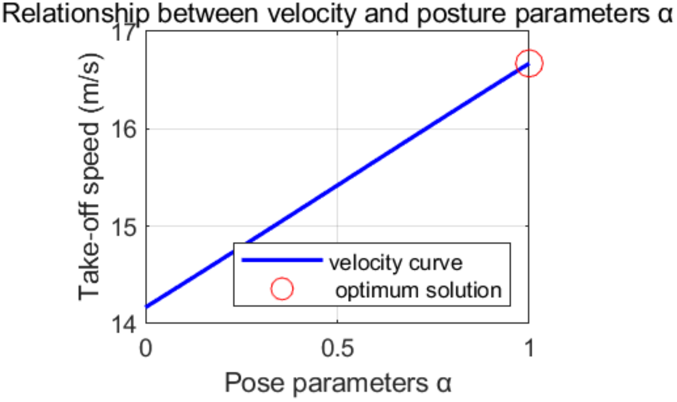

**Figure 3.** Relationship between velocity and posture parameters $\alpha$

It can be seen that with the posture parameter $\alpha$ approaching 1, the maximum take-off speed also increases. That is to say, with the continuous lowering of the athlete's center of gravity, the final maximum take-off speed of the athlete increases, and finally when the athlete is fully squatting, the take-off speed reaches the maximum value.

The variation of speed with time in different cases of $\boldsymbol{\alpha}$ is shown in Figure 4:

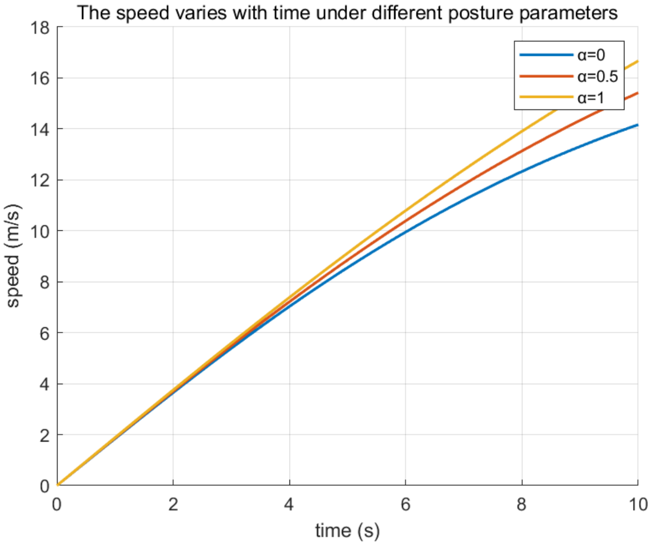

**Figure 4.** Variation of velocity with time under different posture parameters

It can be seen that with the increasing of posture parameter α, the speed of the athlete at the same time also increases. Therefore, naturally, when the athlete takes a posture of α=1 when jumping, the take-off speed is the largest.

The variation of various parameters with $\boldsymbol{\alpha}$ is shown in Figure 5:

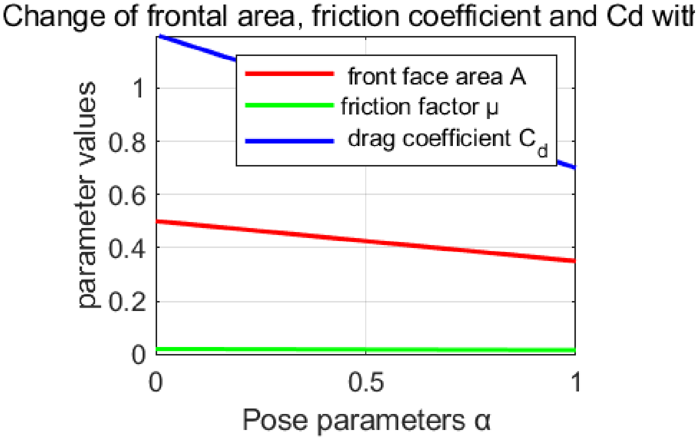

**Figure 5**. Wind area, friction coefficient and $C_d$ change with α

When the posture parameter α is close to 1, the frontal area, friction coefficient and air resistance coefficient are decreasing continuously, which is the dynamic change condition we assume from the actual situation. And this condition is closely related to our final conclusion.

Figure 6, 7 and 8 show the influence of different frontal area, different friction coefficient and different air resistance coefficient on the velocity when α=0.5:

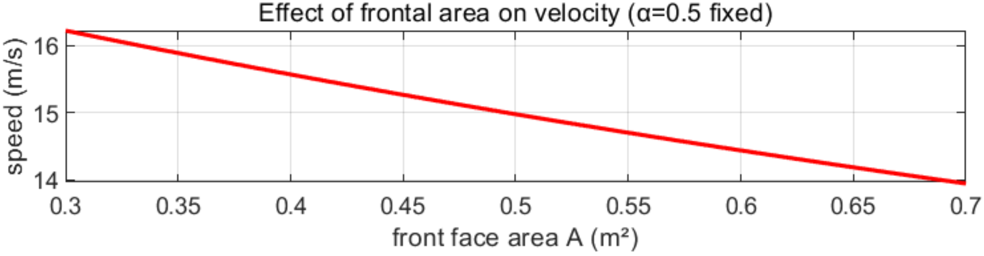

**Figure 6.** The effect of frontal area on velocity

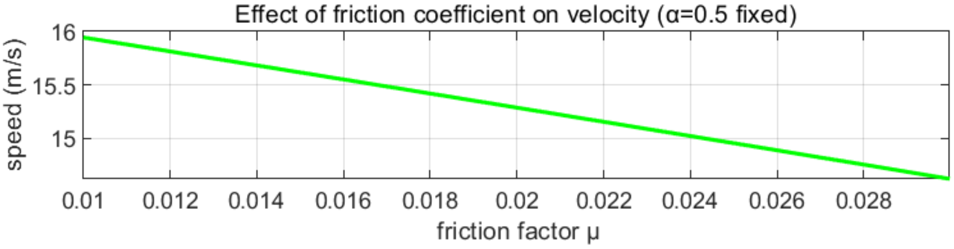

**Figure 7.** Effect of friction coefficient on velocity

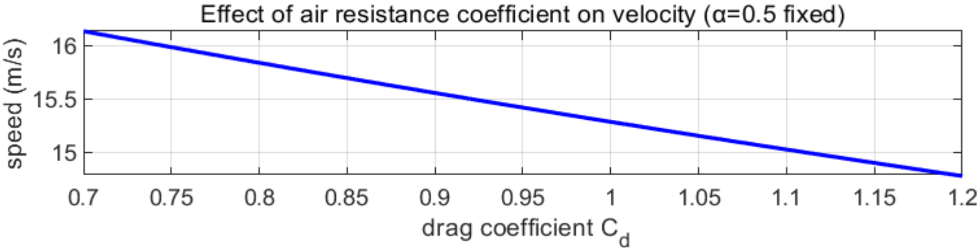

**Figure 8.** Effect of air drag coefficient on velocity

By analyzing the impact of frontal area, friction coefficient, and air resistance coefficient on speed, it can be observed that when α is fixed at 0.5, the final take-off speed decreases as the frontal area, friction coefficient, and air resistance coefficient increase. From Figure 5, we can see that the frontal area, friction coefficient, and air resistance coefficient decrease as α increases. Therefore, when α reaches 1, the frontal area, friction coefficient, and air resistance coefficient are at their minimum, resulting in the maximum speed, which occurs when the athlete assumes a full squatting position. According to Figure 5, the frontal area, friction coefficient, and air resistance coefficient are all relatively small at this point, meaning that in the squatting position, these factors are smaller. Based on actual research, the frontal area is smaller in the squatting position compared to the standing position, with concentrated force points and a lower dynamic friction coefficient. Additionally, due to aerodynamic effects, the air resistance coefficient is also smaller in the squatting position. Thus, in practice, adopting a squatting position results in smaller frontal area, friction coefficient, and air resistance coefficient compared to the standing position, which aligns with the conclusions drawn in this paper. Therefore, when the athlete assumes a full squatting position, the speed ultimately reaches its maximum value.

## 5. Conclusions

In this paper, a dynamic model is established for the acceleration process of athletes in the ski jump on the ski slope, and the ODE solver is combined with the particle swarm optimization algorithm to obtain the conclusion that athletes can minimize air resistance and get the maximum speed when squatting, which is consistent with life experience. The ski jump model established in this paper is not only applicable to the dynamic processes of skiing but can also be extended to the analysis and research of diving, gliding, and other sports. The model presented in this paper is a relatively simple demonstration. In the future, with further consideration of aerodynamic effects and the introduction of dynamic real-time data, machine learning, and cross-domain expansion, the model can become one of the core tools for snow and ice research training, equipment design, and event analysis.

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