A Study on Stair Wear Prediction Based on Archard's Equation and Gaussian Simulation

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Abstract. Ancient stone stairs provide tangible evidence of historical human activity, where their wear patterns reflect how people used and moved through these spaces over time. To quantitatively analyze the relationship between wear depth and usage frequency, this study proposes a stair wear prediction model based on Archard's wear equation. The model integrates key factors such as usage frequency, material hardness, and wear coefficients. Using laser scanning and photogrammetry data, this study optimized the model parameters via least squares and genetic algorithms, obtaining the spatial distribution of wear across a 12×4 stair grid. Further analysis of pedestrian flow patterns revealed that ascending movement causes lighter, edge-focused wear, while descending movement results in deeper, centralized wear due to gravitational effects. By modeling short-term intensive and long-term distributed pedestrian usage with Gaussian distribution functions, this study simulated the corresponding wear characteristics and evaluated them using uniformity, concentration, and smoothness indices. The results demonstrate that high-frequency usage leads to deeper, smoother, and more centralized wear, whereas low-frequency usage produces shallower and more dispersed wear. These findings provide a reliable method for reverse inferring usage patterns of ancient stairs and offer valuable guidance for digital archaeology and heritage conservation.

Keywords: Wear Depth, Usage Frequency, Archard Equation, Pedestrian Flow Patterns.

1. Introduction

Ancient stone steps, as integral components of cultural heritage, encapsulate valuable information about historical human activities, behavioral patterns, and material usage. Their wear characteristics not only reflect the frequency and nature of human interactions but also provide insights into the physical properties and evolution of construction materials, offering significant value to archaeological research and heritage conservation. In recent years, advancements in 3D laser scanning and machine vision technologies have propelled research in the analysis of wear on stone cultural relics. For instance, Meng et al. employed terahertz non-destructive testing combined with support vector machine models to predict deterioration levels in the Yungang Grottoes, enhancing the scientific and proactive aspects of cultural relic preservation [1]. Additionally, Liu et al. introduced a crack prediction method based on wavelet correlation analysis and least squares support vector machines, achieving precise predictions of crack degrees in immovable cultural relics and providing theoretical support for preventive conservation strategies [2]. However, existing studies predominantly focus on superficial correlations between static morphological data and pedestrian flow, lacking integrated modeling that encompasses material mechanical parameters, multi-source data fusion, and dynamic behavioral evolution, thereby limiting the explanatory power and predictive accuracy of wear analyses [3, 4].

To address these limitations, this study proposes a hybrid modeling approach based on Archard's wear equation, optimized through genetic algorithms and spatially simulated using Gaussian distribution methods. By incorporating key factors such as usage frequency, material hardness, wear coefficient, and walking path patterns, the model quantitatively links stair wear distribution with human activity modes. High-precision 3D scanning data, divided into a 12×4 grid with ±0.1mm accuracy, were used for fine-scale wear modeling and inversion. The least squares method combined with genetic algorithm optimization significantly enhanced the spatial accuracy and physical consistency of the wear predictions. Finally, this study simulated the evolution of wear under scenarios of short-term intensive usage and long-term dispersed usage, uncovering the influence of gravity and behavioral pathways on wear distribution. This research offers a novel analytical paradigm for digital archaeology and provides quantitative support for the "minimum intervention" principle in heritage conservation [5].

2. Analysis of Stair Usage Frequency

2.1. Data Description

In this study, the wear depth at each position of the ancient stairs was obtained through laser scanning and photogrammetry.The material hardness and wear coefficients were determined by combining experimental measurements with literature data [6]. To provide an intuitive understanding of the spatial distribution of wear, the collected wear depth data were visualized using three-dimensional and two-dimensional heat maps, as shown in Figure 1. These visualizations reveal significant variations in wear depth across different locations on the stairs, serving as a crucial foundation for the subsequent modeling process.

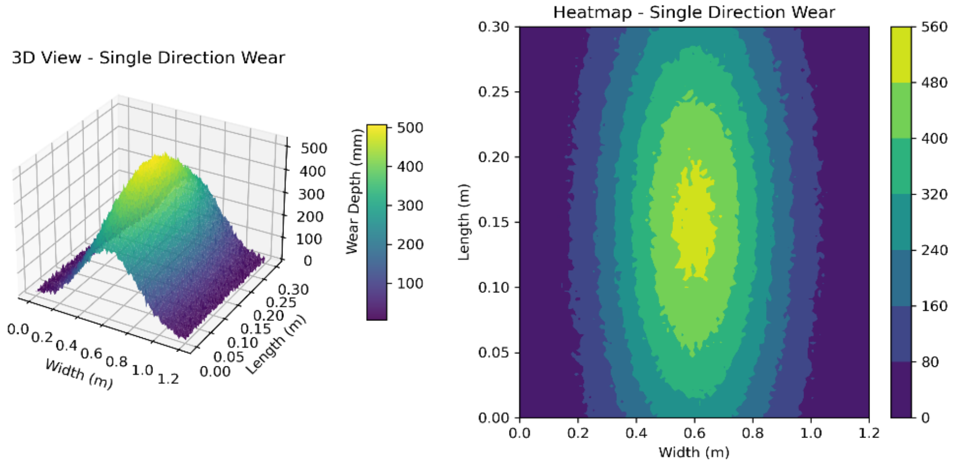

Figure 1. Heatmaps of Wear Depth

2.2. The Establishment of Model

To predict the frequency of stair usage, defined as the number of people passing through per hour, this study developed a predictive model based on Archard's wear equation to establish a quantitative relationship between wear depth and usage frequency [7]. According to Archard's equation, the wear depth of the stairs can be expressed as:

$$d(x,y) = \frac{k \cdot F \cdot f(x,y)}{H} \tag{1}$$

To simplify the model, this study define a comprehensive coefficient:

$$\alpha = \frac{k \cdot F}{H} \tag{2}$$

Considering the presence of measurement errors in reality, the model can be expressed as:

$$d(x,y) = \alpha \cdot f(x,y) + \varepsilon(x,y) \tag{3}$$

The wear depth data obtained through laser scanning is used to infer the usage frequency:

$$f(x,y) = \frac{d(x,y) - \varepsilon(x,y)}{\alpha} \tag{4}$$

2.3. The Solution of Model

2.3.1 Least Squares Method

The least squares fitting method was employed in this study to minimize the error between the model predictions and the observed data. The objective function is defined as follows:

$$\min_{f(x,y)} \iint_R \left(d(x,y) - \alpha \cdot f(x,y)\right)^2 dxdy \tag{5}$$

To prevent overfitting, this study incorporate a regularization term into the objective function [8]:

$$\min_{f(x,y)} \iint_R \left(d(x,y) - \alpha \cdot f(x,y)\right)^2 + \lambda \iint_R f(x,y)^2 dxdy \tag{6}$$

Where λ is the regularization strength parameter, used to balance model complexity and fitting accuracy. Given that the data are discrete, the integral was discretized into a summation form, resulting in the following expression for the objective function:

$$MSE(f) = \frac{1}{n}\sum_{i=1}^{n} \left(d(x_i, y_j) - \alpha \cdot f(x_i, y_j)\right)^2 \tag{7}$$

2.3.2 Genetic Algorithm (GA)

To address the optimization problem for such discrete data, this study utilized a genetic algorithm, which simulates the process of natural selection to iteratively optimize the directional matrix solution [9]. Initially, a population of candidate solutions for the directional matrix $v(x, y)$ was randomly generated. The fitness function was defined as the sum of squared errors:

$$Fitness = \sum_{x,y} \left(d(x,y) - \alpha \cdot v(x,y) \cdot f(x,y)\right)^2 \tag{8}$$

The best-performing individuals were then selected as parents for crossover operations, while random mutations were applied to increase population diversity. Through multiple generations of iteration, the solution of the directional matrix was continuously refined [10].

Finally, two-dimensional heat maps were used to visualize both the actual wear depth and the predicted distribution of usage frequency, as illustrated in Figures 2 and 3.

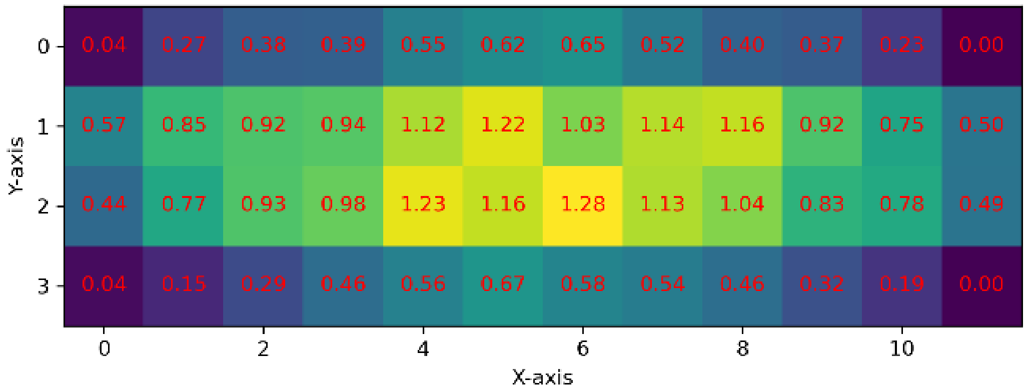

Figure 2. Actual Wear Depth

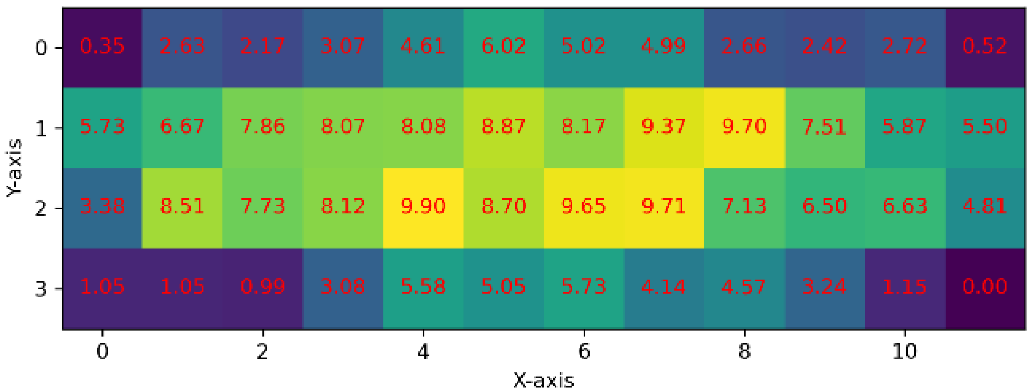

Figure 3. Estimiated Usage Frequency

As shown in Figure 3, the stairs were divided into a 12×4 grid, where the usage frequency of each grid block is visualized through color mapping and numerical values. This visualization enables the inference of pedestrian flow intensity at different locations on the stairs.

3. Analysis of Pedestrian Flow Patterns

3.1. Analysis of Pedestrian Movement Direction

In this study, the wear depth at each position of a double-flight stair was obtained through laser scanning and photogrammetry. Based on the collected data, an analysis of pedestrian movement directions was conducted to explore the influence of pedestrian flow patterns on stair wear distribution. The results indicate that during upward movement, pedestrians typically walk closer to the edges of the stair, maintaining lighter steps and a more stable center of gravity. In contrast, when descending, the influence of gravitational acceleration leads to increased vertical forces exerted on the stair surface. Consequently, more severe wear occurs along the central path of the stair, where pedestrians tend to concentrate [11]. phenomenon is illustrated in Figure 4.

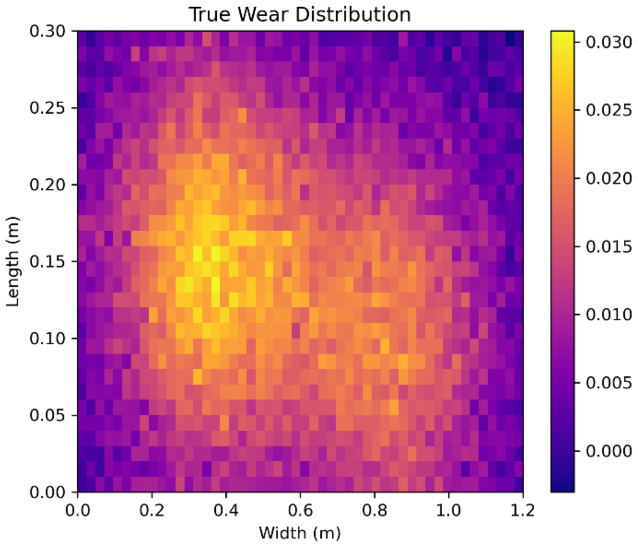

Figure 4. The actual wear distribution map for upward and downward movement

By reviewing relevant literature and examining from a physics perspective, it was found that the vertical force exerted by individuals on the stair differs during upward and downward movement. Studies have shown that when ascending stairs, the impact force on the stair is relatively small, typically ranging from 1.0 to 1.2 times the body weight. When descending stairs, the vertical impact force exerted by a person on the stair surface can reach approximately 1.3 to 1.6 times their body weight due to gravitational acceleration.By combining the Archard wear equation, this study set $\beta_{\text{down}} = 1.33$ as an empirical amplification factor. The model can be expressed as:

$$D_{model}(x,y) = \alpha \cdot f_{up}(x,y) + \beta_{down} \cdot \alpha \cdot f_{down}(x,y) + \epsilon \tag{9}$$

Where $f(x,y)$ is assumed to follow a single Gaussian distribution [12]:

$$\begin{cases} f_{\boldsymbol{up}}(x,y) = \exp\left(-\frac{(x-\mu_{\boldsymbol{up}})^2}{2\sigma_x^2} - \frac{(y-\mu_{y1})^2}{2\sigma_y^2}\right) \\ f_{down}(x,y) = \exp\left(-\frac{(x-\mu_{down})^2}{2\sigma_x^2} - \frac{(y-\mu_{y2})^2}{2\sigma_y^2}\right) \end{cases} \tag{10}$$

To fit the simulated wear depth $D_{\text{model}}(x,y)$ to the actual measured data $D_{\text{true}}(x,y)$, this study use a genetic algorithm to optimize the model parameters:

$$MSE = \frac{1}{N}\sum(D_{model}(x,y) - D_{true}(x,y))^2 \tag{11}$$

Where $D_{\text{true}}(x,y)$ is obtained from the measured data. The genetic algorithm was implemented using Python, iteratively optimizing the parameters to obtain the best-fit solution. The optimized results were visualized using both 2D and 3D heatmaps, as shown in Figure 5.

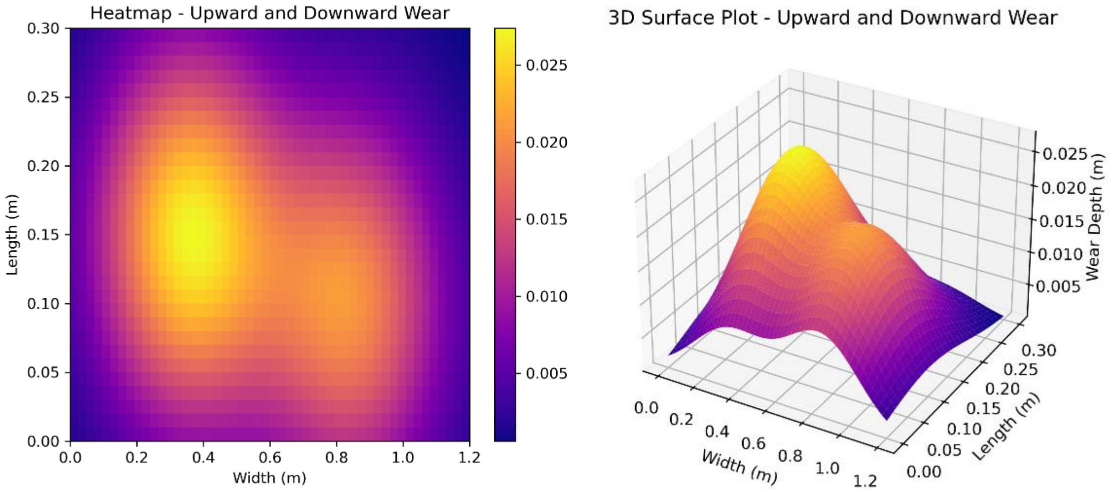

Figure 5. Optimized wear distribution map

To further analyze pedestrian flow patterns, bar charts were used to visualize the traffic intensity at different stair positions for both upward and downward movements. As shown in Figure 6, the distribution patterns align with typical pedestrian behaviors, where the number of ascending and descending pedestrians remains relatively balanced [13].

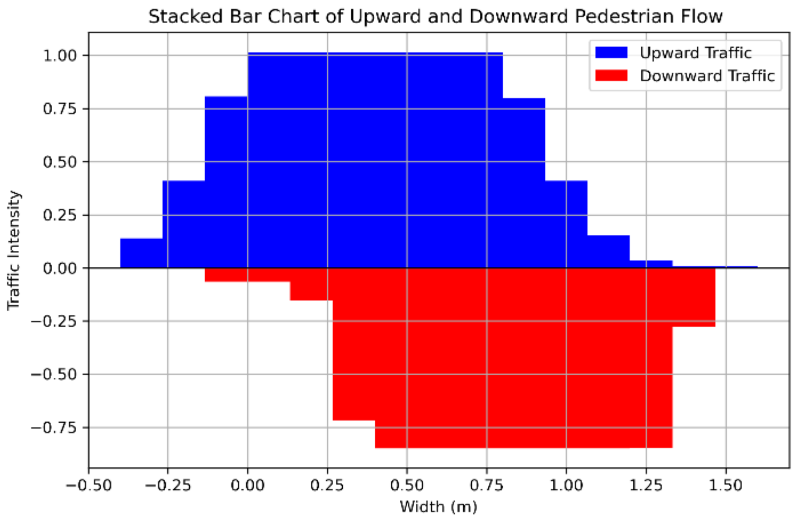

Figure 6. Upward and downward pedestrian flow distribution bar chart

To quantitatively assess the correlation between the simulated data and the actual wear data, the Pearson correlation coefficient R was calculated, defined as:

$$R = \frac{\sum(D_{model}(x,y)-\bar{D}_{model})(D_{true}(x,y)-\bar{D}_{true})}{\sqrt{\sum(D_{model}(x,y)-\bar{D}_{model})^2}\cdot\sqrt{\sum(D_{true}(x,y)-\bar{D}_{true})^2}} \quad (12)$$

A correlation coefficient R close to 1 indicates a strong positive correlation and high model reliability, while a value close to 0 suggests poor correlation and ineffective optimization. The results demonstrate that R approaches 1, indicating that the proposed model effectively simulates the actual wear conditions of the stair under both upward and downward pedestrian flow. This model provides valuable insights for further applications in stair usage analysis and preservation planning.

3.2. Frequent Short-Term Usage vs. Infrequent Long-Term Useage

The Gaussian distribution, also known as the normal distribution, is widely utilized in the modeling of physical phenomena due to its ability to effectively describe centralized and gradually decaying distributions. Based on the fundamental wear depth equation (3), this study model and simulate the impact of pedestrian flow patterns—frequent short-term use versus infrequent long-term use—on the wear situation. This study modify the frequency equation and use a unimodal Gaussian distribution to model and simulate the wear pattern [14]:

$$f(x,y) = c_{min} + c_{max} \cdot e^{-\frac{(x-x_c)^2+(y-y_c)^2}{\tau^2}} + \mathcal{N}(0,\sigma_\epsilon^2) \tag{13}$$

Where c_{max} represents the maximum usage frequency. A larger c_{max} indicates concentrated wear (short-term heavy use), while a smaller c_{min} represents dispersed wear (long-term light use).

To quantitatively evaluate the model’s predictions,three indicators were used to evaluate wear characteristics: uniformity, concentration, and smoothness. Simulation results show that short-term heavy usage leads to deeper and smoother wear patterns, while long-term light usage results in shallower, more uneven wear with rougher surfaces.

The simulation results, as illustrated in Figures 7 and 8 and summarized in Table 1, demonstrate a clear distinction in wear characteristics under different usage patterns. Short-term heavy usage leads to deeper, more centralized, and smoother wear, while long-term light usage produces shallower and more dispersed wear. These findings validate the model’s effectiveness in capturing the mechanical and spatial differences caused by varying pedestrian flow intensities.

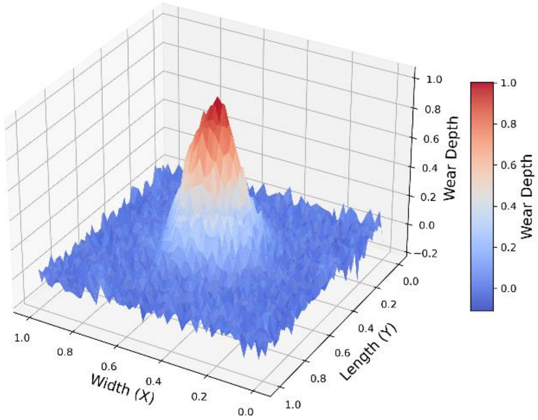

Figure 7. Short-term heavy usage

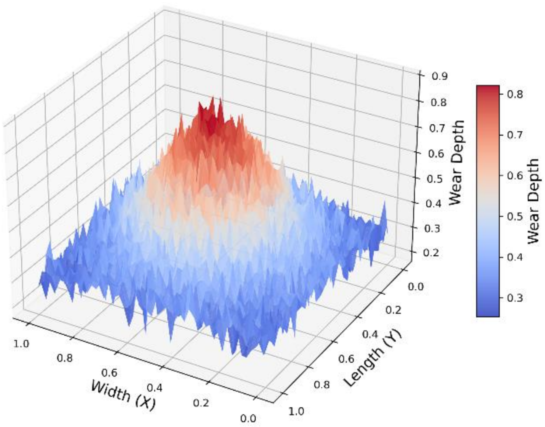

Figure 8. Long-term light usage

Table 1. Numerical values of the indicators obtained from the simulation predictions

Mod	Uniformity Coefficient	Concentration Coefficient	Smoothness
Short-Term	2.743642	0.980890	0.053449
Long-Term	0.182362	0.293204	0.050419

4. Conclusion

In this study, a stair wear prediction model was developed based on Archard's wear equation, integrating laser scanning and photogrammetry data to establish a quantitative relationship between wear depth and usage frequency. The results show that wear is primarily concentrated in high-traffic central areas, following a Gaussian-like distribution. Analysis of pedestrian movement directions revealed that ascending flows tend to cause minor wear along the stair edges, while descending flows, influenced by gravitational acceleration, result in deeper wear in the central region. Furthermore, simulations of short-term heavy usage and long-term light usage demonstrated the significant impact of usage patterns on wear morphology. Short-term heavy usage leads to concentrated, smoother wear surfaces, whereas long-term light usage results in more dispersed, shallow, and rougher wear patterns. This study provides a scientific basis for analyzing stair usage behaviors and offers valuable insights for heritage conservation, particularly in understanding wear origins and formulating restoration strategies.

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