The application of the Maximum Likelihood Estimation principle in communication systems

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Abstract. This paper examines the application of Maximum Likelihood Estimation (MLE) in digital communication systems, with particular emphasis on Binary Phase Shift Keying (BPSK) modulation. The discussion opens with an overview of MLE fundamentals and its asymptotic properties—consistency and efficiency—that establish its suitability for optimal signal detection. The central contribution lies in the derivation of an MLE-based detection algorithm for BPSK transmission over an Additive White Gaussian Noise (AWGN) channel, yielding a transparent optimal decision rule. To substantiate the theoretical framework, a full-scale MATLAB R2024b simulation was developed and executed. The simulation encompasses the complete baseband BPSK transceiver: pulse shaping, carrier modulation, AWGN channel impairment, coherent demodulation, and matched filtering. Performance assessment involves comparing empirically measured Bit Error Rate (BER) against theoretical predictions across varying signal-to-noise ratios. The findings indicate that empirical BER approaches the theoretical bound as the number of simulated bits grows, confirming both the validity and practical efficacy of the derived MLE algorithm. These results underscore the continued relevance of MLE in communication signal detection and furnish a concrete reference for both instructional and engineering contexts.

Keywords: Maximum Likelihood Estimation (MLE); Binary Phase Shift Keying (BPSK); Communication Systems; Bit Error Rate (BER); MATLAB Simulation; Signal Detection; Additive White Gaussian Noise (AWGN) Channel.

1. Introduction

The communication system deeply penetrates every link of modern production and life. From high-speed 5G, industrial Internet collaboration, to the Internet of Things, we can see that. It supports the operation of the digital economy. It is also an important link in ensuring people's livelihood services. Now the hot spots, such as telemedicine, online education, and emergency scheduling rely on real-time transmission, all have high requirements for the speed and reliability of communication systems. In addition, the communication system can also promote global cooperation, and efficient communication can break space-time barriers, making cross-border trade and cross-border exchanges more convenient.

Maximum likelihood estimation is one of the basic methods of parameter estimation. On the premise of known observation data, find a group of parameter values to maximize the probability of observing the current data under this parameter. Firstly, it is asymptotically unbiased and consistent. When the sample size is large enough, the estimated value will be infinitely close to the real parameters. Secondly, MLE is asymptotically efficient and can reach the lowest variance, lower bound and the highest precision among all unbiased estimates. In addition, estimators usually have an asymptotic normal distribution, which is convenient for constructing confidence intervals and hypothesis testing.

Maximum likelihood estimation is often used in various links of communication systems, including shift keying. Shift keying is a modulation technology in digital communication, which refers to the use of digital signals to control a certain parameter of the carrier, such as amplitude, frequency or phase. Its core function is to convert the digital bit stream into analog signals suitable for transmission in the channel, so as to effectively use the band resources and improve the anti-interference ability. The maximum likelihood estimation completes the parameter estimation in signal conversion. Common shift keying includes amplitude shift keying (ASK), frequency shift keying (FSK), phase shift keying (PSK), quadrature amplitude modulation (QAM), etc.

BPSK is a basic form of phase shift keying, which has simple structure and strong anti noise performance. Its core principle can be extended to more complex modulation modes such as QPSK and QAM. Therefore, BPSK is very typical and representative in communication system research.

This paper aims to explore the theory and specific application of MLE in communication systems. The performance of MLE is verified by analyzing various keying theories and simulating BPSK in Matlab.

2. Literature Review

Maximum likelihood estimation has been widely explored, showing its diversity and effectiveness in communication systems. Kaminsky et al. [1] emphasized the fundamental role of the maximum likelihood principle in prediction tasks, pointing out its ability to optimize the likelihood function based on observation data. This shows that MLE mathematical tools have irreplaceable importance in signal analysis of communication systems.

Maximum likelihood estimation is also the theoretical basis of new signal processing methods. In the field of direction of arrival (DOA) estimation, Stoica et al. [2] believe that MLE is directly derived from the statistical characteristics of the original observation data. They pointed out that the maximum likelihood method has excellent performance, but its computation may be large. Therefore, they proposed a compromise method MODE-1. This method has the effect of approximate maximum likelihood estimation while maintaining computational simplicity. It is on the basis of the application of maximum likelihood estimation. It is organically improved according to its practical application in signal processing as well

Chylla et al. [3] developed a general theoretical framework to further expand the application of the maximum likelihood principle. This framework is used to estimate NMR parameters such as frequency and amplitude from multidimensional nuclear magnetic resonance (NMR) data. Their work shows how maximum likelihood can be used to analyze complex multidimensional data sets. This highlights the wide application of maximum likelihood estimation beyond traditional communication systems.

MLE also plays a crucial role in specific communication scenarios Bensley et al. [4] applied maximum likelihood estimation to multi-user CDMA systems. They use basic theory to estimate parameters such as delay, amplitude and phase. In this way, the multi-user estimation problem can be ingeniously decomposed into a single user problem. This shows the reliability of maximum likelihood when facing real users. Similarly, Frantzeskakis et al. [5] used the maximum likelihood in the phase domain for carrier recovery of wireless TDMA systems. In this communication scenario, they extended the method to the recovery method based on mode and decision guidance.

Olson's research [6] reflects the diversity of MLE. He uses probabilistic maximum likelihood technology for mobile robot self localization, which shows the relevance of this principle in navigation and autonomous systems. This is also reflected in the research of Duan et al. [7] They applied maximum likelihood to financial modeling, demonstrating the adaptability of this method to economic data analysis. Hooda et al. [8] discussed the optimization method based on entropy as an alternative or supplementary method for parameter estimation. This enriches the theory of maximum likelihood in a broader context.

MLE is also used in dynamic communication systems. Shi et al [9] introduced event triggered maximum likelihood state estimation and adapted the classical maximum likelihood method to systems with event driven data updates. In addition, Zhang et al [10] studied the maximum likelihood estimation for recovering joint distribution from aggregated data. They considered multiple parameter models such as Gaussian distribution and mixed distribution. This extends the application of maximum likelihood to industrial and statistical environments.

In general, maximum likelihood estimation is the basic principle of communication theory. MLE promotes parameter estimation, signal processing, system synchronization and data analysis in different applications, highlighting its important role in promoting communication principles [1-10].

3. Theoretical Foundation of Maximum Likelihood Estimation

3.1. Definition of MLE

The likelihood function, denoted as $L(\theta)$, is defined as a given set of observed data $x = (x_1, x_2, \ldots, x_n)$ as their joint probability density function, but treated as a function of the parameter θ rather than the data. For independent observations, it is the product of the individual probability densities: $L(\theta) = p(x;\theta) = \prod_{i=1}^{n} p(x_i;\theta)$. The log-likelihood is $\ell(\theta) = \log L(\theta)$.[12]

The Maximum Likelihood Estimate (MLE), denoted $\hat{\theta}(x)$, is the specific value of the parameter θ that maximizes the likelihood function $L(\theta)$ for the fixed observed data x. Equivalently, it maximizes the log-likelihood $\ell(\theta)$.[12] Intuitively, the MLE is the parameter value that makes the observed data most probable or "likely."

3.2. The Maximum of the Function

To find the maximum of a function, we need to conduct these steps:

1. Compute the derivative $\frac{dL}{d\theta}$ and find all the stationary points, i.e., all θ where $\frac{dL}{d\theta} = 0$.

2. Compute the second derivative $\frac{d^2L}{d\theta^2}$. Test the second derivative at each stationary point. A stationary point is a (local) maximum if the second derivative is negative: $\frac{d^2L}{d\theta^2} < 0$.

3. If the interval is closed, then check the value of L at the endpoints and compare them to the values at the local maxima. The greatest value will be the maximum of L.

3.3. Properties

The Maximum Likelihood Estimator (MLE) possesses several desirable asymptotic properties under general regularity conditions.

1. Consistency: As the sample size approaches infinity, the MLE converges in probability to the true parameter value. This is its most fundamental desirable property.

2. Asymptotic Unbiasedness: When the sample size is sufficiently large, the expected value of the MLE is approximately equal to the true parameter value, though it may be biased in small samples.

3. (Asymptotic) Efficiency: Among all consistent and asymptotically normal estimators, the asymptotic variance of the MLE attains the Cramér-Rao lower bound, making it the asymptotically minimum variance unbiased estimator.

4. Asymptotic Normality: As the sample size increases, the distribution of the MLE converges to a normal distribution, centered at its meaning, with variance given by the inverse of the Fisher information matrix. This provides the theoretical foundation for constructing confidence intervals and hypothesis testing.

The received signal in the communication system usually contains noise and information to be demodulated. By establishing the signal model, the information $s(t)$ to be demodulated is regarded as the parameter to be estimated. Using MLE, based on the received signal y, find the s value that maximizes the conditional probability $p(y|s)$ as the decision output.

4. Digital Modulation and MLE-based Detection Algorithm

4.1. Shift Keying of Digital Modulation

The application of MLE in digital modulation is very typical. Digital modulation is the process of converting digital baseband signals into analog signals suitable for channel transmission. It changes the amplitude, frequency or phase of the high-frequency carrier to make its changes correspond to the digital bit stream. The common shift keying methods are as follows:

1. Amplitude shift keying

ASK transmits digital information by changing the amplitude of the carrier signal. Generally, carrier amplitude is used to represent binary "1", and none or minimum amplitude is used to represent "0". It is simple to implement but has weak anti noise and anti attenuation ability. It is mainly used for low-speed data transmission, such as early optical communication and RFID tags [11].

2. Frequency shift keying

FSK transmits information by changing the frequency of the carrier signal. For example, the frequency f1 represents "1", and f2 represents "0". Its advantages are strong anti amplitude interference ability, relatively simple implementation, but low spectral efficiency. It is widely used in wireless paging, low-speed modem and some wireless sensor networks.[11]

3. Phase shift keying

PSK transmits information by changing the phase of carrier signal. The most basic BPSK uses 0° and 180° phases to represent "0" and "1" respectively. PSK achieves higher anti noise performance and spectral efficiency than ASK and FSK under the condition of constant envelope, which is the cornerstone of modern communication. The evolution forms of QPSK, 8PSK, etc. improve the transmission rate by increasing the phase state.[11]

4. Quadrature amplitude modulation

QAM is a modulation technology combining amplitude and phase. It performs independent ASK modulation on two orthogonal carriers, so that more bits can be transmitted on the same frequency. For example, 16QAM can transmit 4-bit information in a symbol cycle. QAM achieves the best balance between extremely high spectral efficiency and anti noise performance, so it becomes the core modulation technology of high-speed wired and wireless communication, such as Wi Fi, 5G. [11].

4.2. BPSK modulation principle and system model

Next, we will take BPSK as an example to illustrate its modulation principle and system model composition.

Let us set the pair of signals to represent binary symbols 1 and 0, among these $f_c = \frac{n_c}{T_b}$:

$s_1(t) = \frac{\sqrt{2E_b}}{\sqrt{T_b}}\cos(2\pi f_c t), 0 < t \le T_b$ for symbol 1.

$s_2(t) = \frac{\sqrt{2E_b}}{\sqrt{T_b}}\cos(2\pi f_c t + \pi) = -\frac{\sqrt{2E_b}}{\sqrt{T_b}}\cos(2\pi f_c t),\ 0 < t \le T_b$ for symbol 0.

Let us set the basis function as $\phi_1(t) = \sqrt{\frac{2}{T_b}}\cos(2\pi f_c t), 0 < t \le T_b$.

Satisfying $s_1(t) = \sqrt{E_b}\phi_1(t), 0 < t \le T_b$; $s_2(t) = -\sqrt{E_b}\phi_1(t), 0 < t \le T_b$.

In the signal-space diagram of BPSK, we calculate:

$$s_{11} = \int_0^{T_b} s_1(t)\,\phi_1(t)dt = \sqrt{E_b} \tag{1}$$

$$s_{21} = \int_0^{T_b} s_2(t)\,\phi_1(t)dt = -\sqrt{E_b} \tag{2}$$

Then we apply the maximum likelihood decision rule, divide the signal phase into two regions Calculate the observation element:

$$x_1 = \int_0^{T_b} x(t)\,\phi_1(t)dt \tag{3}$$

When symbol 0 is sent, the conditional PDF of X1:

$$f_{X1}(x_1|0) = \frac{1}{\sqrt{\pi N_0}}\exp\left[-\frac{(x_1 - s_{21})^2}{N_0}\right] = \frac{1}{\sqrt{\pi N_0}}\exp\left[-\frac{(x_1 + \sqrt{E_b})^2}{N_0}\right] \tag{4}$$

The conditional probability of the receiver deciding in favor of symbol 1, given that symbol 0 was transmitted:

$$p_{10} = \int_0^{\infty} f_{X1}(x_1|0)\,dx_1 = \frac{1}{\sqrt{\pi N_0}}\int_0^{\infty}\exp\left[-\frac{(x_1 + \sqrt{E_b})^2}{N_0}\right]dx_1 = \frac{1}{\sqrt{\pi}}\int_{\sqrt{\frac{E_b}{N_0}}}^{\infty}\exp(-z^2)dz = \frac{1}{2}\text{erfc}(\sqrt{\frac{E_b}{N_0}}) \tag{5}$$

Among which $\text{erfc}(x) = 2Q(\sqrt{2}x)$, $Q(x)$ is the right tail of Normal Distribution.

Then, the average probability of symbol error for BPSK (BER) is [11]:

$$p_e = p_0 p_{10} + p_1 p_{01} = \frac{1}{2}\text{erfc}(\sqrt{\frac{E_b}{N_0}}) \tag{6}$$

5. Matlab Simulation Design and Results Analysis

5.1. Simulation Process

This experiment will use MatlabR2024b for simulation. The main goal of this simulation is to verify the performance of MLE based BPSK detection algorithm, mainly the bit error rate (BER) performance. The specific steps are as follows:

1. Initialization and parameter setting

Set the key parameters of the system, including sampling frequency Fs, symbol rate Rb, carrier frequency fc, and calculate the oversampling rate.

Defines the signal-to-noise ratio range of the simulation.

Initialize the array, store the empirical bit error rate obtained from subsequent calculation, and call the berawgn function to calculate the theoretical bit error rate as a comparison benchmark.

2. Generate transmission signal

Generate a random binary bit sequence and map it into bipolar symbols according to BPSK rules.

Three pulse shaping filter options (rectangular, sinc, root rising cosine RRC) are available for the modulation process

BPSK symbols are upsampled and then convolved with the designed pulse shaping filter to generate a continuous baseband waveform.

Multiply the baseband signal by the cosine carrier, complete the spectrum shift, and obtain the band-pass modulated BPSK signal.

3. Analog channel

In a cycle, traverse all the set Es/N0 values.

Calculate the required SNR (dB) according to the current signal-to-noise ratio Es/N0 and oversampling rate.

Finally, the awgn function is used to add additive white gaussian noise to the modulated signal, which is the key to simulate the bit error rate of the real channel.

4. Receiving and demodulation (receiving end)

Perform coherent demodulation, multiply the local carrier with the same frequency and phase with the modulated signal containing noise, and down convert it back to the baseband.

The matched filter is used to filter the demodulated signal to maximize the output signal-to-noise ratio.

Then the output of the matched filter is sampled at symbol rate, that is, down sampling.

Finally, hard decisions (positive decisions are+1, negative decisions are -1) is made according to the positive and negative of the sampling value to recover the sent BPSK symbol sequence.

5. Performance evaluation and analysis

At each SNR, compare the recovered symbol sequence with the original transmitted symbol sequence, count the number of errors, and calculate the empirical bit error rate.

Finally, the theoretical BER curve is drawn and compared with the empirical BER curve obtained by simulation.

In the simulation process, the primary intermediate outputs include the original BPSK constellation, the pulse-shaped baseband waveform, the carrier-modulated waveform, and the power spectral density of the modulated signal. These outputs are integrated in Figure 1 to support in-text verification of the modulation procedure.

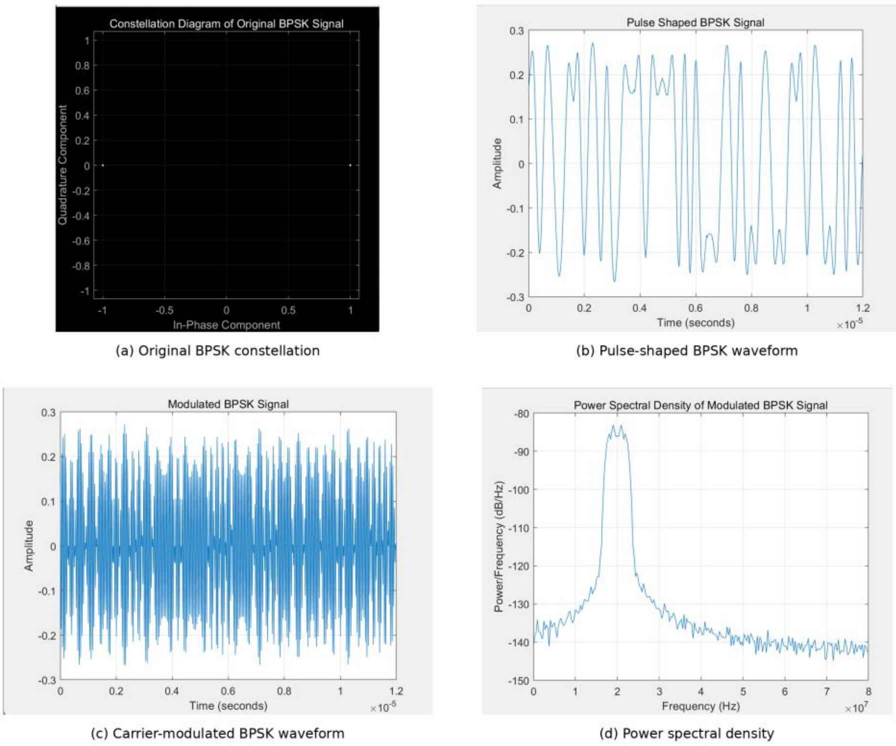

Figure 1. Intermediate outputs of the MATLAB-based BPSK simulation: (a) original BPSK constellation; (b) pulse-shaped BPSK waveform; (c) carrier-modulated BPSK waveform; (d) power spectral density of the modulated signal.

The full MATLAB implementation has been removed from the manuscript body and provided separately as supplementary material.

5.2. Simulation results and analysis

This is the curve obtained by simulation. There are a theoretical curve and a simulation curve in the figure.

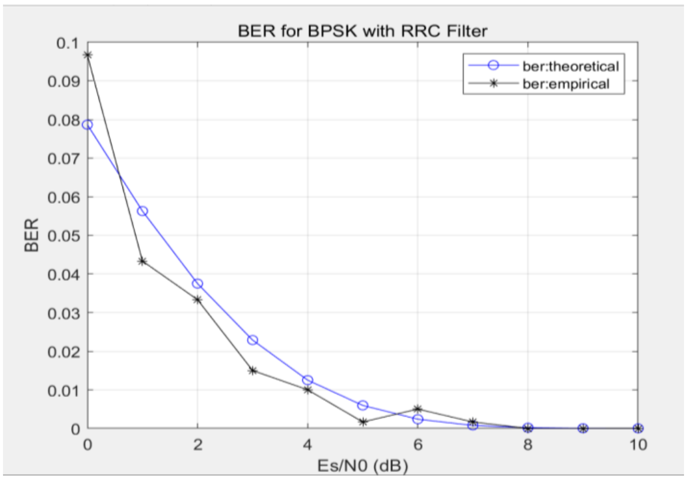

Figure 2. BER simulation result with nums = 60.

It can be seen from the figure that when the number of bits is 60, the trends of the two curves are the same, but the curve shapes differ greatly.

This is because nums are set too small. We changed nums to 600, 6000, 6e4 and 6e5 respectively, and resimulated them.

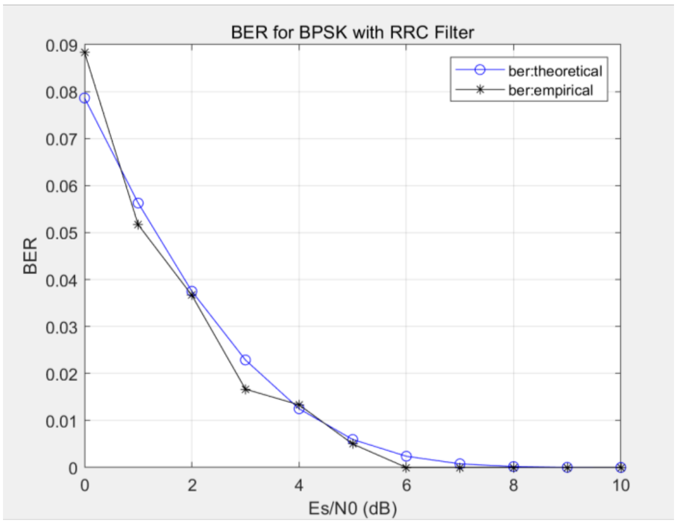

Figure 3. BER simulation result with nums = 600.

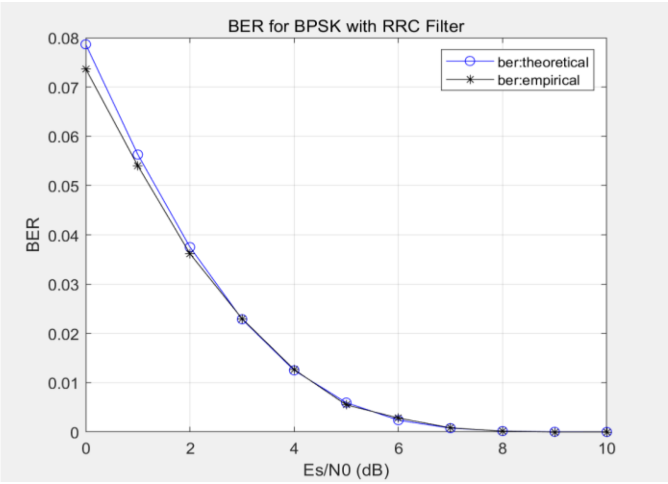

Figure 4. BER simulation result with nums = 6000.

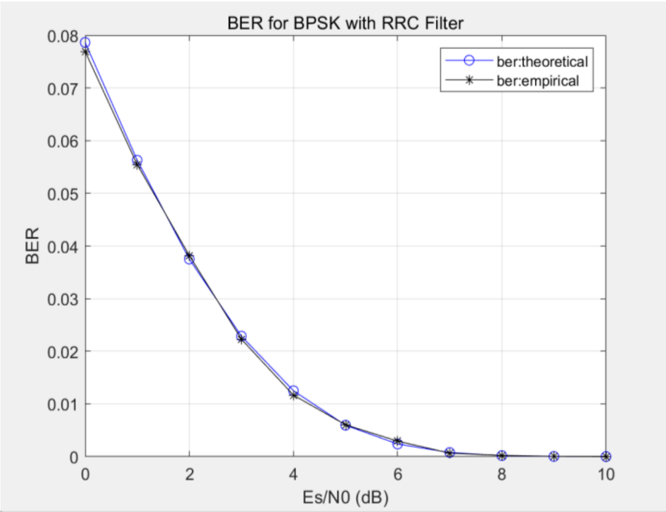

Figure 5. BER simulation result with nums = 6e4.

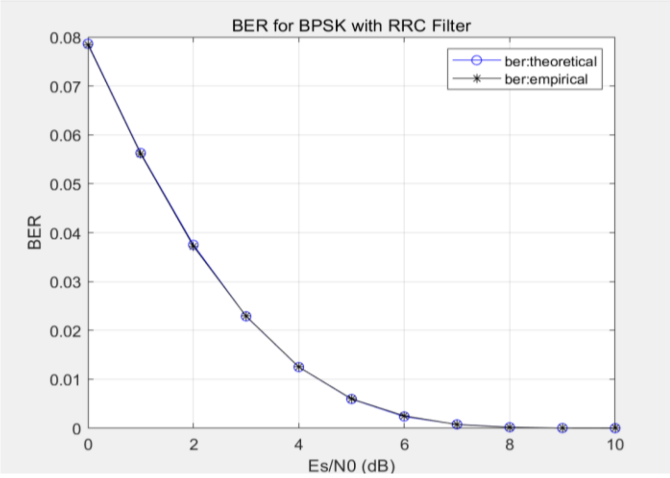

Figure 6. BER simulation result with nums = 6e5.

At this time, we can see that with the increase of nums, the two curves tend to coincide gradually.

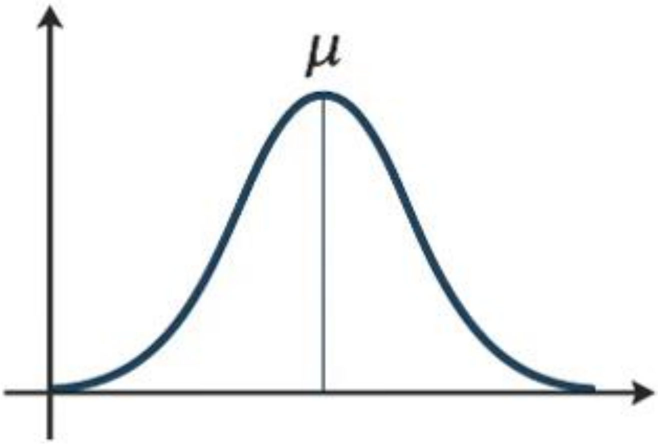

Figure 7. Normal distribution.

Comparing the normal distribution curve in Figure 7 with the BER expression, it can be seen that the BER corresponds to the right-tail probability of the normal distribution.

Therefore, this experiment verifies the application of MLE algorithm in BPSK.

6. Conclusion

6.1. Limitations and Extensions

At present, this research focuses on BPSK system simulation under additive white Gaussian noise channel. In order to expand the breadth and depth of the research, it can be carried out from different aspects in the future.

First, on the system model, simulation can be extended to higher-order modulation methods (such as QPSK, 16QAM, etc.) to study their performance under high frequency spectral efficiency, and fading channels (such as Rayleigh fading, Rice fading) and multipath channel models that are more suitable for the actual wireless environment can be introduced to evaluate the robustness of the system under complex channels.

Secondly, at the algorithm implementation level, we can deeply analyze the complexity bottleneck of maximum likelihood estimation implemented on embedded platforms or dedicated hardware, and explore their low-power, low latency optimization methods and approximate computing strategies.

6.2. Research Conclusions and Significance

This paper focuses on the application of maximum likelihood estimation theory in communication systems

Firstly, the basic principle of MLE and its core idea in signal detection are systematically introduced, that is, to find the most likely original signal to be sent based on the received signal samples. Then, the theory is focused on the basic binary phase shift keying system model. Under the assumption of additive white Gaussian noise channel, a signal detection algorithm based on MLE criterion is derived.

In order to verify the theoretical analysis, a complete Matlab simulation experiment is designed. Under AWGN channel conditions, the bit error rate performance curves of the MLE detection algorithm under different signal-to-noise ratios are obtained. The empirical BER curve obtained from the simulation is compared with the theoretical BER formula of the classical coherent BPSK demodulation, and the results show that they are highly consistent, which strongly verifies the correctness and practical effectiveness of the deduced algorithm.

Through the complete process from theoretical derivation to simulation verification, this study reaffirmed that the maximum likelihood estimation, as a basic and powerful statistical tool, has certain application value. The paper provides a clear case for understanding the specific implementation of MLE in digital communication and has reference significance for teaching demonstration of related courses and algorithm feasibility assessment in engineering practice.

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